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Axisymmetric Modes of Cylindrical Resonators with Cascaded Inhomogeneous Dielectrics

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Abstract— A generic numerical scheme is developed to calculate the resonant frequency of axisymmetric modes in an inhomogeneous cylindrical dielectric resonator. The resonator consists of sections of cylindrically stratified dielectrics within a cylindrical waveguide. In each section, the TM_{0m} and TE_{0m} waveguide modes are solved by expanding the H_ϕ and E_ϕ components in terms of the eigenmodes in an empty waveguide. The fields in each section are then expanded in terms of these TM_{0m} and TE_{0m} modes. The transverse resonance technique is then applied to obtain the resonant frequencies. Comparison with literatures validates the effectiveness of this approach. Results with continuous dielectric profiles are also obtained.

I. INTRODUCTION

Cylindrical cavities have been used to cure materials [1], to measure complex permittivity of materials [2], and as a resonator in microwave circuits [3]–[11]. In all these applications, the circular waveguide section forming the cavity contains inhomogeneous dielectrics. For the resonator application, the resonant frequencies of

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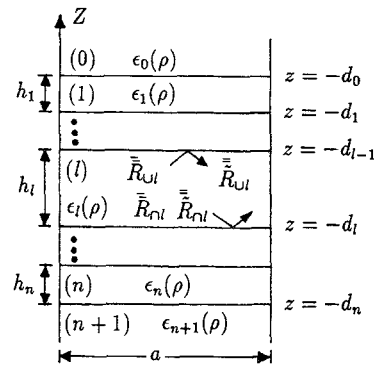


Fig. 1. Geometrical configuration of cascaded circular waveguide sections loaded with inhomogeneous dielectrics.

the dielectric loaded cavity need to be determined precisely. Several coupled dielectric rod or ring resonators can be arranged coaxially in a circular waveguide to form a bandpass filter. The resonant frequencies of the axisymmetric $TE_{01\delta}$ and $TM_{01\delta}$ modes have been calculated by using a mode-matching technique [3], [4]. For both modes, the resonant frequencies are below the cutoff frequency of the TE_{01} waveguide mode. In [5] and [6], the resonant frequency of nonaxisymmetric hybrid modes are calculated by using a similar technique.

In [7], a finite integration technique (FIT) based on the integral forms of Maxwell's equations is proposed to calculate the resonant frequencies of a cavity filled with an inhomogeneous dielectric. A brief summary of mode nomenclature is also provided in [7]. In [8], a variational expression is used to calculate the resonant frequencies of axisymmetric modes where the radial variation of field components are expanded by the first-order finite element (FE) basis functions and the axial variation is expanded in terms of sinusoids. Finite-difference method in the frequency domain [9], finite-difference time-domain (FDTD) method [10], and finite element method (FEM) [11] have also been used.

Mode-matching method proves to be efficient for many canonical resonator structures. For example, the cylindrical dielectric rod and ring in a cylindrical cavity. Usually, the eigenmodes in a stratified medium need to be solved first to represent the field distribution in the later stage. If the dielectric ring consists of many layers or if a dielectric rod has a continuous permittivity profile, conventional mode-matching method becomes tedious or impossible. For such structures, finite element method, FDTD method, and FIT method can be used at the expense of finer grids to express the fields accurately.

In this paper, we will present a generic numeric scheme to solve such problems. First, the eigenmodes in each uniform dielectric loaded waveguide section are obtained by solving a symmetric eigenvalue problem, where dielectrics with continuous profile can also be handled. Reflection matrices at the junctions of waveguide sections are defined to reduce the number of unknowns. Then the transverse resonance technique is applied to obtain the resonant frequencies of the resonators.

II. FORMULATION

Fig. 1 shows the configuration of a cylindrical resonator with radius a , which consists of several sections of circular waveguides loaded with inhomogeneous dielectrics. The permittivity in each layer is a piecewise continuous functions of ρ and is independent of ϕ and z .

Axisymmetric modes exist in such a medium, and are categorized into TM (to z) and TE (to z) modes [7]. For the TM modes, the existing field components are E_z , E_ρ , and H_ϕ . For the TE modes, the existing field components are H_z , H_ρ , and E_ϕ .

First, consider the TM modes in an infinitely long circular waveguide with an inhomogeneous dielectric profile which is uniform along the axial direction. Expand Maxwell's equations to obtain

$$\begin{aligned} E_\rho &= \frac{1}{i\omega\epsilon} \frac{\partial H_\phi}{\partial z}, E_z = -\frac{1}{i\omega\epsilon} \frac{1}{\rho} \frac{\partial}{\partial \rho} \rho H_\phi \\ \left(\rho\epsilon \frac{\partial}{\partial \rho} \frac{1}{\rho} \frac{\partial}{\partial \rho} + k^2 + \frac{\partial^2}{\partial z^2} \right) \rho H_\phi &= 0. \end{aligned} \quad (1)$$

Next, express H_ϕ of the n th eigenmodes by a set of basis functions $S_m(\rho)$ as

$$\begin{aligned} H_{n\phi} &= \phi_n(\rho) e^{\pm i k_{nz} z} \\ &= \sum_{m=1}^N b_{nm} S_m(\rho) e^{\pm i k_{nz} z} \\ &= \bar{b}_n^t \cdot \bar{S}(\rho) e^{\pm i k_{nz} z} \end{aligned} \quad (2)$$

where $\bar{b}_n^t = [b_{n1}, \dots, b_{nN}]$, and $\bar{S}^t(\rho) = [S_1(\rho), \dots, S_N(\rho)]$. Choose the H_ϕ distribution in an empty circular waveguide as the basis functions, i.e., $S_m(\rho) = J_1(\xi_m \rho/a)$ with $J_0(\xi_m) = 0$. Substitute (2) into (1), then take the inner product of $S_p(\rho)$ with the resulting equation and use the integration by parts technique to have

$$\begin{aligned} \sum_{m=1}^N b_{nm} \left[-\left\langle \frac{1}{\rho\epsilon} \frac{\partial}{\partial \rho} \rho S_p(\rho) \frac{\partial}{\partial \rho} \rho S_m(\rho) \right\rangle + \langle S_p(\rho), \omega^2 \mu \rho S_m(\rho) \rangle \right] \\ = k_{nz}^2 \sum_{m=1}^N b_{nm} \langle S_p(\rho), \rho \epsilon^{-1} S_m(\rho) \rangle, 1 \leq p \leq N \end{aligned} \quad (3)$$

where the inner product is defined over $0 \leq \rho \leq a$. Hence, (3) constitutes a symmetric eigenvalue problem to be solved numerically for the propagation constant k_{nz} . The eigenvectors \bar{b}_n 's satisfy the orthonormality specification that $\bar{b}_p^t \cdot \bar{N} \cdot \bar{b}_m = \delta_{pm}$ where $\bar{N} = \langle \bar{S}(\rho), \rho \epsilon^{-1} \bar{S}^t(\rho) \rangle$. The general field in the waveguide can be expressed in terms of these eigenmodes as

$$H_\phi = \sum_{n=1}^N \phi_n(\rho) e^{\pm i k_{nz} z} \alpha_n = \bar{\phi}^t(\rho) \cdot e^{\pm i \bar{K}_z z} \cdot \bar{\alpha}$$

$$E_\rho = \sum_{n=1}^N \pm \frac{k_{nz}}{\omega\epsilon} \phi_n(\rho) e^{\pm i k_{nz} z} \alpha_n = \pm \frac{1}{\omega\epsilon} \bar{\phi}^t(\rho) \cdot \bar{K}_z \cdot e^{\pm i \bar{K}_z z} \cdot \bar{\alpha} \quad (4)$$

where $\bar{\phi}^t(\rho) = [\phi_1(\rho), \dots, \phi_N(\rho)]$, $\bar{K}_z = \text{diag}[k_{1z}, \dots, k_{Nz}]$, and $e^{\pm i \bar{K}_z z} = \text{diag}[e^{\pm i k_{1z} z}, \dots, e^{\pm i k_{Nz} z}]$.

Next, use the waveguide modes to represent the fields in layer (l) of the resonator as

$$\begin{aligned} H_{l\phi} &= \bar{\phi}_l^t(\rho) \cdot \left[e^{i \bar{K}_{lz} z_l} \cdot \bar{\alpha}_l + e^{-i \bar{K}_{lz} z_l} \cdot \bar{\beta}_l \right] \\ E_{l\rho} &= \frac{1}{\omega\epsilon_l} \bar{\phi}_l^t(\rho) \cdot \bar{K}_{lz} \cdot \left[e^{i \bar{K}_{lz} z_l} \cdot \bar{\alpha}_l - e^{-i \bar{K}_{lz} z_l} \cdot \bar{\beta}_l \right] \end{aligned} \quad (5)$$

where $z_l = z + d_l$. The first term in $H_{l\phi}$ and $E_{l\rho}$ is a wave propagating in the $+z$ direction, and the second term is a wave propagating in the $-z$ direction. Define a reflection matrix $\bar{R}_{\cup l}$ at the upper boundary $z = -d_{l-1}$ so that the reflection matrix multiplied by the upward wave gives the downward wave, i.e., $\bar{R}_{\cup l} \cdot e^{i \bar{K}_{lz} h_l} \cdot \bar{\alpha}_l = e^{-i \bar{K}_{lz} h_l} \cdot \bar{\beta}_l$. Define another reflection matrix $\bar{R}_{\cap l}$ at the lower boundary $z = -d_l$ so that the reflection matrix multiplied by the downward wave gives the upward wave, i.e., $\bar{R}_{\cap l} \cdot \bar{\beta}_l = \bar{\alpha}_l$. Thus, we obtain the resonance condition

$$\det(\bar{I} - \bar{R}_{\cap l} \cdot e^{i \bar{K}_{lz} h_l} \cdot \bar{R}_{\cup l} \cdot e^{i \bar{K}_{lz} h_l}) = 0. \quad (6)$$

The resonant frequencies are obtained by solving (6).

The fields in layer (m) with $m > l$ can be expressed as

$$\begin{aligned} H_{m\phi} &= \bar{\phi}_m^t(\rho) \cdot \left[e^{i \bar{K}_{mz} z_m} \cdot \bar{R}_{\cap m} + e^{-i \bar{K}_{mz} z_m} \right] \cdot \bar{\beta}_m \\ E_{m\rho} &= \frac{1}{\omega\epsilon_m} \bar{\phi}_m^t(\rho) \cdot \bar{K}_{mz} \cdot \left[e^{i \bar{K}_{mz} z_m} \cdot \bar{R}_{\cap m} - e^{-i \bar{K}_{mz} z_m} \right] \cdot \bar{\beta}_m. \end{aligned} \quad (7)$$

By matching the boundary conditions that $H_{r\phi} = H_{(r+1)\phi}$ and $E_{r\rho} = E_{(r+1)\rho}$ at $z = -d_r$, we have

$$\begin{aligned} (\bar{R}_{\cap r} + \bar{I}) \cdot \bar{\beta}_r &= \bar{B}_{r(r+1)} \\ &\cdot \left[e^{i \bar{K}_{(r+1)z} h_{r+1}} \cdot \bar{R}_{\cap(r+1)} + e^{-i \bar{K}_{(r+1)z} h_{r+1}} \right] \cdot \bar{\beta}_{r+1} \\ (\bar{R}_{\cap r} - \bar{I}) \cdot \bar{\beta}_r &= \bar{K}_{rz}^{-1} \cdot \bar{B}_{(r+1)r}^t \cdot \bar{K}_{(r+1)z} \\ &\cdot \left[e^{i \bar{K}_{(r+1)z} h_{r+1}} \cdot \bar{R}_{\cap(r+1)} - e^{-i \bar{K}_{(r+1)z} h_{r+1}} \right] \cdot \bar{\beta}_{r+1} \end{aligned} \quad (8)$$

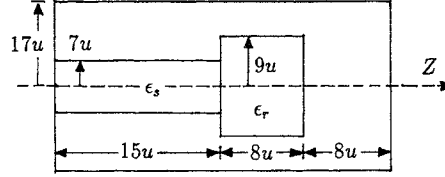
$$\begin{aligned} \bar{R}_{\cap r} &= \left\{ \left[\bar{I} - e^{i \bar{K}_{(r+1)z} h_{r+1}} \cdot \bar{R}_{\cap(r+1)} \cdot e^{i \bar{K}_{(r+1)z} h_{r+1}} \right]^{-1} \cdot \left(\bar{K}_{rz}^{-1} \cdot \bar{B}_{(r+1)r}^t \cdot \bar{K}_{(r+1)z} \right)^{-1} \right. \\ &\quad \left. + \left[\bar{I} + e^{i \bar{K}_{(r+1)z} h_{r+1}} \cdot \bar{R}_{\cap(r+1)} \cdot e^{i \bar{K}_{(r+1)z} h_{r+1}} \right]^{-1} \cdot \bar{B}_{r(r+1)}^{-1} \right\}^{-1} \\ &\cdot \left\{ \left[\bar{I} - e^{i \bar{K}_{(r+1)z} h_{r+1}} \cdot \bar{R}_{\cap(r+1)} \cdot e^{i \bar{K}_{(r+1)z} h_{r+1}} \right]^{-1} \cdot \left(\bar{K}_{rz}^{-1} \cdot \bar{B}_{(r+1)r}^t \cdot \bar{K}_{(r+1)z} \right)^{-1} \right. \\ &\quad \left. - \left[\bar{I} + e^{i \bar{K}_{(r+1)z} h_{r+1}} \cdot \bar{R}_{\cap(r+1)} \cdot e^{i \bar{K}_{(r+1)z} h_{r+1}} \right]^{-1} \cdot \bar{B}_{r(r+1)}^{-1} \right\} \end{aligned} \quad (9)$$

$$\begin{aligned} H_{m\phi} &= \bar{\phi}_m^t(\rho) \cdot \left[e^{i \bar{K}_{mz} z_m} + e^{i \bar{K}_{mz} (h_m - z_m)} \cdot \bar{R}_{\cup m} \cdot e^{i \bar{K}_{mz} h_m} \right] \cdot \bar{\alpha}_m \\ E_{m\rho} &= \frac{1}{\omega\epsilon_m} \bar{\phi}_m^t(\rho) \cdot \bar{K}_{mz} \cdot \left[e^{i \bar{K}_{mz} z_m} - e^{i \bar{K}_{mz} (h_m - z_m)} \cdot \bar{R}_{\cup m} \cdot e^{i \bar{K}_{mz} h_m} \right] \cdot \bar{\alpha}_m \end{aligned} \quad (10)$$

$$\begin{aligned} (\bar{I} + \bar{R}_{\cup r}) \cdot e^{i \bar{K}_{rz} h_r} \cdot \bar{\alpha}_r &= \bar{B}_{r(r-1)} \cdot \left[\bar{I} + e^{i \bar{K}_{(r-1)z} h_{r-1}} \cdot \bar{R}_{\cup(r-1)} \cdot e^{i \bar{K}_{(r-1)z} h_{r-1}} \right] \cdot \bar{\alpha}_{r-1} \\ (\bar{I} - \bar{R}_{\cup r}) \cdot e^{i \bar{K}_{rz} h_r} \cdot \bar{\alpha}_r &= \bar{K}_{rz}^{-1} \cdot \bar{B}_{(r-1)r}^t \cdot \bar{K}_{(r-1)z} \\ &\cdot \left[\bar{I} - e^{i \bar{K}_{(r-1)z} h_{r-1}} \cdot \bar{R}_{\cup(r-1)} \cdot e^{i \bar{K}_{(r-1)z} h_{r-1}} \right] \cdot \bar{\alpha}_{r-1} \end{aligned} \quad (11)$$

TABLE I
COMPARISON OF RESONANT FREQUENCIES (GHZ) COMPUTED BY USING THIS APPROACH WITH THOSE IN [7], N IS THE NUMBER OF BASIS FUNCTIONS, $\epsilon_r = 37.3$, $\epsilon_s = 3.78$, $u = 0.427$ mm

Mode	$N = 10$	$N = 15$	$N = 20$	$N = 25$	$N = 30$	[7], measured	[7], computed
TE ₀₁	7.02	7.02	7.02	7.02	7.02	7.037	6.943
TE ₀₂	11.39	11.39	11.39	11.39	11.39	11.391	11.316
TM ₀₁	9.54	9.45	9.43	9.42	9.41	9.296	9.185
TM ₀₂	11.44	11.30	11.27	11.25	11.23	11.113	10.943



where \bar{B}_{ij} is defined as $\langle \rho \epsilon_i^{-1}(\rho) \bar{\phi}_i(\rho), \bar{\phi}_j^t(\rho) \rangle$. A recursive formula is thus obtained as shown in (9) at the bottom of the previous page.

The fields in layer (m) with $m < l$ can be expressed as shown in (10) at the bottom of the previous page. By matching the boundary conditions that $H_{r\phi} = H_{(r-1)\phi}$ and $E_{r\rho} = E_{(r-1)\rho}$ at $z = -d_{r-1}$, we have as shown in (11) at the bottom of the previous page. A recursive formula is thus obtained as shown in (12) at the bottom of the page.

The same procedure can be applied to the TE modes. First, consider the TE modes in an infinitely long circular waveguide with an inhomogeneous dielectric profile which is uniform along the axial direction. Expand Maxwell's equations in terms of H_z , H_ρ , and E_ϕ . Next, expand the eigenmodes of E_ϕ by a set of basis functions $\tilde{S}_m(\rho)$, and choose the E_ϕ distribution in an empty circular waveguide as the basis functions to expand E_ϕ . The eigenmodes can be obtained by solving the eigenvalue problem formed by taking the inner product of $\tilde{S}_p(\rho)$ with the equation satisfied by E_ϕ .

Next, use the waveguide modes to represent the fields in layer (l) of the resonator. Reflection matrices \bar{R}_{ul} and \bar{R}_{rl} are defined to relate the upward wave and the downward wave in layer (l). Finally, the resonant frequencies are obtained by solving the resonance condition

$$\det \left(\bar{I} - \bar{R}_{rl} \cdot e^{i\bar{K}_{lz}h_l} \cdot \bar{R}_{ul} \cdot e^{i\bar{K}_{lz}h_l} \right) = 0. \quad (13)$$

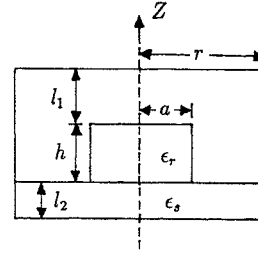
Recursive formulas for \bar{R}_{ul} and \bar{R}_{rl} can be derived by matching E_ϕ and H_ρ at interfaces between contiguous layers.

III. NUMERICAL RESULTS

First, we show the resonant frequencies of the TE₀₁, TE₀₂, TM₀₁, and TM₀₂ modes of a cylindrical dielectric-loaded resonator as shown in Table I. The results from [7] are also shown for comparison.

TABLE II
COMPARISON OF RESONANT FREQUENCIES (GHZ) COMPUTED BY USING THIS APPROACH WITH THOSE IN THE REFERENCES, $N = 30$, $\epsilon_r = 38$, $\epsilon_s = 2.33$, $a = h = 4$ mm, $r = 16$ mm, $l_1 = 3$ mm, $l_2 = 0.79$ mm, [X] : [4] IN [8]

Mode	present	[8]	[X]
TE ₀₁	6.82	6.82	6.73
TE ₀₂	10.75	10.79	-
TE ₀₃	12.02	12.09	12.10
TM ₀₁	9.52	9.51	9.40
TM ₀₂	13.13	13.10	-



The convergence rate for the TE₀₁ and TE₀₂ modes are faster than that for the TM₀₁ and TM₀₂ modes in this case.

Table II shows the resonant frequencies of a dielectric resonator on top of a substrate as in a circuit board environment. Our results compare favorably with those in the literatures. Table III shows the resonant frequencies of a cylindrical dielectric resonator. The results are close to those in the literatures.

Next, we calculate the resonant frequencies of two symmetrically coupled dielectric ring resonators in a circular waveguide. The permittivity of the ring is assumed to have a parabolic profile with an extreme value ϵ_m at $\rho = (a + b)/2$. The resonant frequencies

$$\begin{aligned} \bar{R}_{ur} = & \left\{ \left[\bar{I} - e^{i\bar{K}_{(r-1)z}h_{r-1}} \cdot \bar{R}_{u(r-1)} \cdot e^{i\bar{K}_{(r-1)z}h_{r-1}} \right]^{-1} \cdot \left(\bar{K}_{rz}^{-1} \cdot \bar{B}_{(r-1)r}^t \cdot \bar{K}_{(r-1)z} \right)^{-1} \right. \\ & \left. + \left[\bar{I} + e^{i\bar{K}_{(r-1)z}h_{r-1}} \cdot \bar{R}_{u(r-1)} \cdot e^{i\bar{K}_{(r-1)z}h_{r-1}} \right]^{-1} \cdot \bar{B}_{r(r-1)}^{-1} \right\}^{-1} \\ & \cdot \left\{ \left[\bar{I} - e^{i\bar{K}_{(r-1)z}h_{r-1}} \cdot \bar{R}_{u(r-1)} \cdot e^{i\bar{K}_{(r-1)z}h_{r-1}} \right]^{-1} \cdot \left(\bar{K}_{rz}^{-1} \cdot \bar{B}_{(r-1)r}^t \cdot \bar{K}_{(r-1)z} \right)^{-1} \right. \\ & \left. - \left[\bar{I} + e^{i\bar{K}_{(r-1)z}h_{r-1}} \cdot \bar{R}_{u(r-1)} \cdot e^{i\bar{K}_{(r-1)z}h_{r-1}} \right]^{-1} \cdot \bar{B}_{r(r-1)}^{-1} \right\} \end{aligned} \quad (12)$$

TABLE III

COMPARISON OF RESONANT FREQUENCIES (GHZ) COMPUTED BY USING THIS APPROACH WITH THOSE IN THE REFERENCES, $N = 30$, $\epsilon_r = 35.74$, $h = 7.62$ mm, $l_1 = l_2 = 3.81$ mm, $a = 8.636$ mm, $r = 12.954$ mm, $[Y] : [20]$ IN [8]

Mode	present	[8]	[y]	[9]
TE ₀₁	3.428	3.435	3.428	3.429
TE ₀₂	5.419	5.493	5.462	5.412
TM ₀₁	4.572	4.601	4.551	4.542

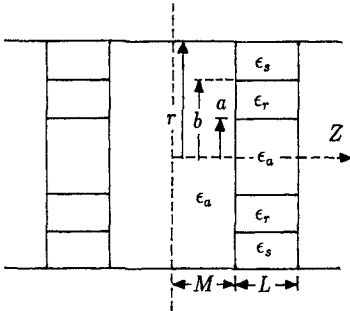
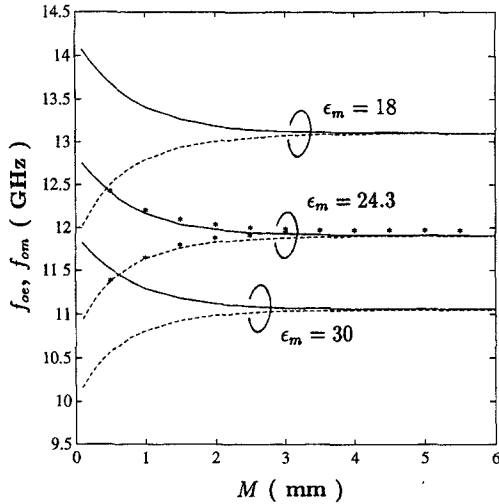
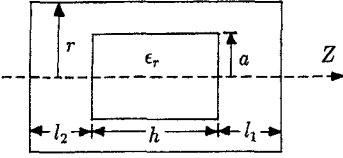


Fig. 2. Resonant frequencies of the TE₀₁₆ mode of two symmetrically coupled dielectric ring resonator, $\epsilon_r(\rho) = 24.3 + 4(\epsilon_m - 24.3)(\rho - a)(b - \rho)/(b - a)^2$, $\epsilon_s = 1.031$, $\epsilon_a = 1$, $b = 2.455$ mm, $a = 0.3b$, $r = 2.39b$, $(b/L)^2 = 0.4625$. —: electric wall in the middle. - - - -: magnetic wall in the middle (results from [3]).

are below the cutoff frequency of the circular waveguide. Due to the structure symmetry, either an electric wall or a magnetic wall can be inserted in the middle plane to form two equivalent problems. The resulting resonant frequency are denoted by f_{oe} (electric wall) and f_{om} (magnetic wall), respectively. The resonant frequencies as a function of the resonator separation are shown in Fig. 2. The results with a flat profile in the ring match well with those in [3]. The resonant frequency decreases as ϵ_m increases. The difference between f_{oe} and f_{om} increases as the two resonators move closer to each other.

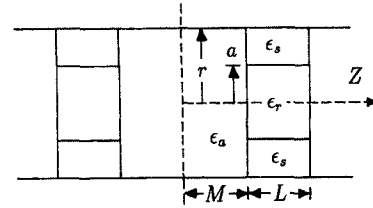
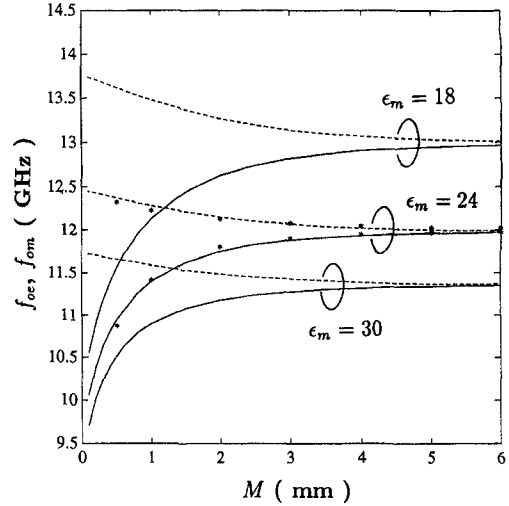


Fig. 3. Resonant frequencies of the TM₀₁₆ mode of two symmetrically coupled dielectric rod resonator, $\epsilon_r(\rho) = 24 + (\epsilon_m - 24)(1 - \rho^2/a^2)$, $\epsilon_s = 1.031$, $a = 3.635$ mm, $r = 5.45$ mm, $L = 4.04$ mm. —: electric wall in the middle. - - - -: magnetic wall in the middle (results from [4]).

Finally, we calculate the resonant frequencies of two symmetrically coupled dielectric rod resonators in a circular waveguide. The permittivity of the rod is assumed to have a parabolic profile with an extreme value ϵ_m at $\rho = 0$. The resonant frequencies are below the cutoff frequency of the circular waveguide. As shown in Fig. 3, the results with a flat profile in the rod match well with those in [4]. The resonant frequency decreases as ϵ_m increases. The difference between f_{oe} and f_{om} increases as the two resonators move closer to each other. Note that f_{oe} is lower than f_{om} in this case, and f_{oe} is higher than f_{om} for the coupled dielectric rings in the previous case.

IV. CONCLUSION

A general numeric scheme combining the eigenvalue method and the transverse resonance technique has been developed to calculate the resonant frequencies of a cylindrical resonator consisting of cascaded sections of circular waveguides loaded with inhomogeneous dielectrics. The results obtained by using this approach compare favorably with those in the literatures. The resonant frequencies with continuous dielectric profiles have also been calculated, which can not be done by using conventional mode-matching methods.

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Precision Broadband Wavemeter for Millimeter and Submillimeter Range

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Abstract—A precise, broadband, Fabry–Perot wavemeter has been designed and built to measure wavelengths in the millimeter and submillimeter range. The design of the wavemeter is novel in that it enhances the fundamental mode over a wide band and permits determination of the exact longitudinal index of the mode. With the use of an exact mode number in wavelength calculations, high measurement accuracies, to the extent permissible by the quality factor of the resonator, can be obtained. The wavemeter was tested by measuring well-known spectral lines of the OCS molecule in the frequency range of 72–607 GHz. Measurement of 24 OCS lines demonstrated an accuracy of better than 2×10^{-5} in relative units and 0.87×10^{-5} in rms units for frequency/wavelength. A discussion of further development and automation of the wavemeter is included.

I. INTRODUCTION

In short-wave millimeter and submillimeter regions, open resonators of the Fabry–Perot type are analogs to closed cavities of the centimeter and millimeter wave regions [1]. They are based on concepts associated with optical frequencies and so are called quasi-optical Fabry–Perot resonators. The most common resonator employs a curved mirror at one end and a flat mirror at the other end. Stable Gaussian-beam resonances of the TEM_{mnq} type can be supported by these open resonators [2]. High quality factors on the order of 10^5 are routinely possible, which enable sharp resonances and high measurement accuracy of resonance locations.

Even so, the conventional method of measuring wavelength leads to diminished accuracy. It consists of tuning the high- Q quasi-optical Fabry–Perot resonator to two consecutive resonances (two consecutive longitudinal modes) and measuring the difference between the corresponding positions of a movable mirror. The difference is equal to half of the wavelength, with necessary diffraction corrections. The procedure is subject to two main sources of error.

- 1) The measured wavelength is the small difference between the two large distances (on the order of 100 mm) between the mirrors at the q th and $(q+1)$ th longitudinal modes. The relative accuracy of measuring each resonance position is on the order of $1/Q = 10^{-5}$, but the relative accuracy of the difference in distance is on the order of $q/Q = 10^{-3}$.
- 2) If the oscillator whose radiation wavelength is to be measured drifts by 10^{-4} during the time the resonator is tuned from one mode to the another, the error in the wavelength measurement will be $q \times 10^{-4} = 10^{-2}$.

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